

Mass dependence of HBT correlations in e^+e^- annihilation

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Abstract

Mass dependence of the effective source radii, observed in hadronic Z^0 decays by several LEP I experiments, is analyzed in a model which assumes proportionality between four-momentum of a produced particle and the four-vector describing its space-time position at the freeze-out. It is shown that this relation (commonly accepted in description of high-energy collisions) can explain the data, provided all particles are emitted from a "tube" of ~ 1 fm in diameter at a constant proper time ~ 1.5 fm.

1 Introduction

Recently, two of us have pointed out [1] that the generalized Bjorken-Gottfried hypothesis [2], relating the space-time position of a hadron produced in a high-energy collision to its 4-momentum, can qualitatively explain the mass-dependence of the interaction radii observed in e^+e^- annihilation at LEP I [3, 4]. As discussed in detail in [1], this effect is a manifestation of the well-established observation [5] that a correlation between the momentum and the emission point of a particle can drastically affect the results of the HBT correlation experiment. In the present note we want to explore in more detail the idea formulated in [1] in order to verify, if it indeed provides a viable framework for the understanding of the mass dependence of the HBT parameters.

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The generalized Bjorken-Gottfried hypothesis, as formulated in [1], postulates the linear relation between the 4-momentum of the produced particle and the space-time position at which it is produced¹:

$$q_\mu = \lambda x_\mu. \quad (1)$$

Relation (1) implies

$$\lambda = \frac{M_\perp}{\tau} \quad (2)$$

where $M_\perp^2 = E^2 - q_\parallel^2 = m^2 + q_\perp^2$, and

$$\tau = \sqrt{t^2 - z^2} \quad (3)$$

is the *longitudinal* proper time after the collision (t and z are time and longitudinal position of the production point).

Since this picture is purely classical, it represents only a qualitative idea, whose application to the description of the actual data requires an adequate reformulation taking into account the effects of the quantum nature of the system considered. In [1] we have proposed to use (1) and (2) as a guide-line for construction of the source function² $S(P, X)$ [7, 8] related to the density matrix in momentum space by a Fourier transform

$$\rho(q = P + \frac{1}{2}Q, q' = P - \frac{1}{2}Q) = \int d^4X e^{iQX} S(P, X). \quad (4)$$

All variables are four dimensional, so that both space and time integrations are involved. Thus specifying the source function fixes completely the single particle properties of the model.

Similarly as the standard Wigner function [9], $S(P, X)$ gives approximately (as far as possible without contradicting quantum mechanics) the single-particle distribution in momentum and in space-time. Therefore it has an intuitive meaning³. which can be exploited for implementation of the relations (1)-(2).

The construction of $S(P, X)$ (described in the next section) requires specification of several details which can only be determined from a confrontation with data. We thus analyse the single- and two-pion distributions in the framework of the scheme proposed in [1] and compare them with the data from the DELPHI experiment at LEP I [10] in order (i) to pin down the parameters describing the system and (ii) to verify that their values are reasonable, i.e., that

¹To our knowledge, the first application of relation (1) to a discussion of the quantum interference between identical particles was proposed (in a different context) by Csorgo and Zimanyi [6]

²It was called there a "generalized Wigner function".

³It should be realized that, in contrast to the standard Wigner function which relates the particle wave functions at different positions but at *the same time*, the source function relates the particle production amplitudes at different positions and *at different times*. (as is clearly seen from (4)). Consequently some care is needed in order to assess its physical interpretation.

the whole scheme is not unrealistic. Our conclusion is rather optimistic: the proposed scheme seems to describe adequately the LEP I data. As, in addition, it provides a reasonable intuitive picture of the hadronization, one may hope that it represents a useful framework for the description of the general properties of this process.

In the next two sections we describe the construction of the source function and calculate the single particle density matrix in momentum space. In Section 4 the formulae for two-particle correlation functions are derived. Comparison with the data is discussed in Section 5 and 6. The last section contains our conclusions and outlook.

2 The Source Function

To implement the conditions (1)-(2) we postulate the source function in the factorized form

$$S(P, X) = F(\tau) S_{\parallel} S_{\perp} \quad (5)$$

where

$$S_{\parallel} = \exp \left[\frac{1}{2\delta_{\parallel}^2} \left(P_{+} - \frac{M_{\perp}}{\tau} X_{+} \right) \left(P_{-} - \frac{M_{\perp}}{\tau} X_{-} \right) \right] \quad (6)$$

and

$$S_{\perp} = \exp \left[-\frac{X_{\perp}^2}{2r_{\perp}^2} \right] \exp \left[-\frac{\left(\vec{P}_{\perp} - \frac{M_{\perp}}{\tau} \vec{X}_{\perp} \right)^2}{2\delta_{\perp}^2} \right]. \quad (7)$$

Here

$$X_{\pm} = t \pm z; \quad P_{\pm} = P_0 \pm P_z. \quad (8)$$

so that

$$M_{\perp}^2 = P_{+} P_{-}; \quad \tau^2 = X_{+} X_{-}. \quad (9)$$

We have used Gaussian forms in order to simplify the evaluation of the Fourier transform (4). As shown below, this admittedly crude assumption seems sufficient at the present level of analysis and accuracy of the data.

The first exponential factor in S_{\perp} represents a standard⁴ cylindrically symmetric "tube" of radius r_{\perp} in configuration space⁵. The remaining exponential introduces a correlation between the momentum and the emission point of the particle, as required by the generalized Bjorken-Gottfried condition (1). Such correlations are known to influence strongly the HBT effect on particle spectra

⁴For related approaches see, e.g., [11].

⁵To simplify the argument, we ignore the rapidity and z dependence of the single particle spectrum. This seems a reasonable approximation in the central rapidity region at high energy and can be easily removed, if necessary.

[5] and were shown in [1] to induce the mass dependence of the HBT radii. The reason for the mass dependence is easily seen from (2): at fixed τ , the correlation between momentum and position depends on transverse mass.

The parameters δ_{\parallel} and δ_{\perp} parametrize the correlation lengths or - in other words - the size of the region (centered at X_{μ}) from which the observed particles emerge. They are arbitrary, apart from inequalities required for consistency with the quantum uncertainty principle, see, e.g., [14], which put on them some weak lower limits.

Finally, the function $F(\tau)$ gives the distribution of the proper time τ at which the particles are created.

Using now the formula (4) and the formulae of this section one can evaluate the single-particle density matrix in momentum space, which is a necessary step in the evaluation of both the single-particle spectrum and the two-particle HBT correlations. This calculation is outlined in the next section.

3 Density matrix in momentum space

Substituting (5) into (4) we have

$$\rho(q, q') = \int \tau d\tau F(\tau) \rho_{\parallel} \rho_{\perp} \quad (10)$$

where

$$\rho_{\perp} = \int d^2 X_{\perp} S_{\perp} e^{-i \vec{X}_{\perp} \vec{Q}_{\perp}} \quad (11)$$

and

$$\rho_{\parallel} = \int d\eta e^V. \quad (12)$$

Here η is the pseudorapidity:

$$\eta = \log \left(\frac{X_+}{X_-} \right) \quad (13)$$

and

$$V = \frac{1}{2\delta_{\parallel}^2} \left(P_+ - \frac{M_{\perp}}{\tau} X_+ \right) \left(P_- - \frac{M_{\perp}}{\tau} X_- \right) + i(Q_0 t - Q_{\parallel} z). \quad (14)$$

From (4) we also see that

$$P = \frac{1}{2}(q + q'); \quad Q = q - q' \quad (15)$$

The Gaussian integral (11) for ρ_{\perp} can be explicitly performed with the result

$$\rho_{\perp}(\vec{q}_{\perp}, \vec{q}'_{\perp}) = 2\pi r_{eff}^2 \exp \left(-\frac{\vec{P}^2}{2\omega^2} - \frac{\vec{Q}_{\perp}^2 r_{eff}^2}{2} \right) \exp \left[-i \frac{M_{\perp} \tau v^2}{\omega^2} \vec{P}_{\perp} \vec{Q}_{\perp} \right] \quad (16)$$

where

$$\omega^2 = M_\perp^2 v^2 + \delta_\perp^2; \quad v^2 = r_\perp^2 / \tau^2; \quad r_{eff}^2 = \frac{r_\perp^2 \delta_\perp^2}{\omega^2} \quad (17)$$

The longitudinal integral is somewhat more complicated. We first express the energies and longitudinal momenta in terms of the respective rapidities

$$P_\pm = M_\perp e^{\pm Y}; \quad Q_0 = m_\perp \cosh y - m'_\perp \cosh y'; \quad Q_\parallel = m_\perp \sinh y - m'_\perp \sinh y' \quad (18)$$

Substituting this into (14) we obtain after some algebra

$$V = \frac{M_\perp^2}{\delta_\parallel^2} (1 - \cosh(Y - \eta)) + i \frac{\tau}{M_\perp} \left(\vec{P}_\perp \vec{Q}_\perp \cosh(Y - \eta) + m_\perp m'_\perp \sinh(y - y') \sinh(\eta - Y) \right) \quad (19)$$

One sees from this formula that V depends only on $Y - \eta$ and $y - y'$. Consequently, after integration over η the result depends only on $y - y'$.

The integral (12) can now be evaluated. We first change the variable $\eta - Y \rightarrow \xi$. Then we express the hyperbolic functions of ξ by exponentials. Using the integral representation

$$K_0(z) = \int_0^\infty e^{-z \cosh y} dy \quad (20)$$

we obtain

$$\rho_\parallel = 2 \exp \left(\frac{M_\perp^2}{\delta_\parallel^2} \right) K_0(s) \quad (21)$$

with

$$s^2 = \frac{M_\perp^4}{\delta_\parallel^4} - \frac{\tau^2}{M_\perp^2} (\vec{P}_\perp \vec{Q}_\perp)^2 - 2i \frac{\tau M_\perp}{\delta_\parallel^2} \vec{P}_\perp \vec{Q}_\perp + \frac{\tau^2}{4M_\perp^2} m_\perp^2 m'^2_\perp \sinh^2(y - y') \quad (22)$$

Using the identities

$$\begin{aligned} m_\perp \cosh y + m'_\perp \cosh y' &= 2M_\perp \cosh Y \\ m_\perp \sinh y + m'_\perp \sinh y' &= 2M_\perp \sinh Y \end{aligned} \quad (23)$$

we arrive, after some algebra, at our final formula

$$s^2 = \frac{M_\perp^4}{\delta_\parallel^4} - \tau^2 Q_t^2 - i \frac{\tau M_\perp}{\delta_\parallel^2} (m_\perp^2 - m'^2_\perp) \quad (24)$$

where

$$Q_t^2 = Q_0^2 - Q_\parallel^2 \quad (25)$$

This completes the evaluation of the single-particle density matrix.

4 Single-particle distribution and two-particle correlation function

The single particle distribution is given by the diagonal elements of the density matrix. From the formulae of the previous section we thus obtain

$$\rho(q) \equiv \frac{dn}{dyd^2q_\perp} = 2\pi r_\perp^2 \delta_\perp^2 \exp\left(\frac{m_\perp^2}{\delta_\parallel^2}\right) K_0\left(\frac{m_\perp^2}{\delta_\parallel^2}\right) I(q_\perp^2) \quad (26)$$

where

$$I(q_\perp^2) = \int \tau d\tau F(\tau) \bar{\omega}^{-2} \exp\left(-\frac{q_\perp^2}{2\bar{\omega}^2}\right) \quad (27)$$

and

$$\bar{\omega}^2 = m_\perp^2 v^2 + \delta_\perp^2 \quad (28)$$

To obtain information on the two-particle correlation function, one has to make further assumptions. We follow the standard treatment [7, 8, 14], assuming that one can evaluate the two-particle correlation function as if there were no other correlations between particles except for those induced by quantum interference. Under this condition the normalized two-particle correlation function is given by

$$C(q_1, q_2) = \frac{|\rho(q_1, q_2)|^2}{\rho(q_1)\rho(q_2)} \quad (29)$$

The final point we want to discuss is the selection of the variables used for comparison with the data, and the corresponding Jacobians.

The system of two identical particles is fully described by the 6 components of the momenta. The phase-space volume is

$$d\Omega = dy dy' d^2q_\perp d^2q'_\perp = 2\pi dy dy' dP_\perp^2 dQ_\perp^2 d\phi \quad (30)$$

where ϕ is the angle between \vec{P}_\perp and \vec{Q}_\perp .

As seen from the formulae of Section 3, it is convenient to replace the two rapidities by Q_t^2 and Y . Using the identity

$$Q_t^2 = (m_{1\perp} - m_{2\perp})^2 - 4m_{1\perp}m_{2\perp} \sinh^2\left(\frac{y_1 - y_2}{2}\right) \quad (31)$$

one obtains

$$dy_1 dy_2 = \frac{dY dQ_t^2}{\sqrt{Q_t^4 - 2(m_{1\perp}^2 + m_{2\perp}^2)Q_t^2 + (m_{1\perp}^2 - m_{2\perp}^2)^2}}. \quad (32)$$

Thus everything can be expressed in terms of $Y, Q_t^2, Q_\perp^2, P_\perp^2$ and ϕ . It is sometimes convenient, however, to consider other variables. For completeness we give below some kinematic relations which can be useful in data analysis.

$$\mu^2 \equiv 2(m_{1\perp}^2 + m_{2\perp}^2) = 4m^2 + 4P_\perp^2 + Q_\perp^2 \quad (33)$$

$$Q_{out} = |Q_{\perp}| \cos \phi; \quad M_{\perp}^2 = \mu^2 - Q_t^2 \quad (34)$$

$$m_{1\perp}^2 - m_{2\perp}^2 = 2|P_{\perp}||Q_{\perp}| \cos \phi = 2|P_{\perp}|Q_{out} \quad (35)$$

5 Data: Single-particle distribution

Eq.(10) represents the density matrix as an integral over the proper time τ at which the particles are produced. In the present paper, following [1], we shall accept the approximation that the production happens in a very narrow interval of τ , so that the integration over τ simply amounts to introduce a fixed value $\tau = \tau_0$ in the formulae of the previous section. In this way the unknown function $F(\tau)$ is replaced by one parameter, τ_0 . The other three parameters are: v , δ_{\perp} and δ_{\parallel} , each with a very clear physical meaning.

The first step in the data analysis should be the correct description of the single particle spectrum using the Eq.(26). This also allows to restrict somewhat the values of the four parameters we have to our disposal.

For this purpose the data sample of $\approx 3 * 10^5$ Z^0 hadronic decays from the DELPHI experiment was used [10]. Hadronic events have been selected using standard DELPHI cuts [12] which limit the contamination of beam-gas, $\gamma\gamma, \mu\mu$ and ee events to less than 0.1% and $\tau\tau$ contamination to below 0.2%. In this study only two-jet like events were considered ($Thrust \geq 0.95$). The distribution of transverse momentum with respect to the event thrust axis was constructed for tracks which are not obvious decay products of V^0 's and are not positively identified as K's or p's.

The formula (26) has been fitted to the distribution in the q_{\perp}^2 region up to 2.0 GeV^2 which contains 99% of the data. The small $q_{\perp}^2 < 0.020 \text{ GeV}^2$ interval was excluded from fits because it is particularly uncertain experimentally due to the contribution of low momentum pions from the D^* decays and to a possible distortion of the distribution by imperfect approximation of the primary quark direction by the measured thrust axis.

The unknown parameters (v , δ_{\perp} and δ_{\parallel} plus normalization) were determined by minimizing a χ^2 function with the MINUIT program [13]. The parameter τ_0 was kept fixed in fits at the value of 1.5 fm. Since correlations between some fit parameters are high, we used a 3 standard deviation correlation ellipse for each parameter pair to estimate their statistical errors. In addition, the stability of the results of the fit has been checked by varying the range of fitted q_{\perp}^2 . The extreme values of a parameter resulting from these fits were used as a measure of the range of a parameter allowed by the data.

Although the fits converged easily to a deep minimum, their χ^2 value was not always statistically acceptable. Nevertheless, we accept these fits as long as the shape of the experimental distribution is correctly reproduced over a wide range of q_{\perp}^2 . The too high χ^2_{min}/dof value is attributed to underestimated errors (imperfect treatment of systematics, no background subtraction etc) rather, than

to failure of the fitted hypothesis. An example fit is shown in Fig.1. The fit results are summarized in Table 1.

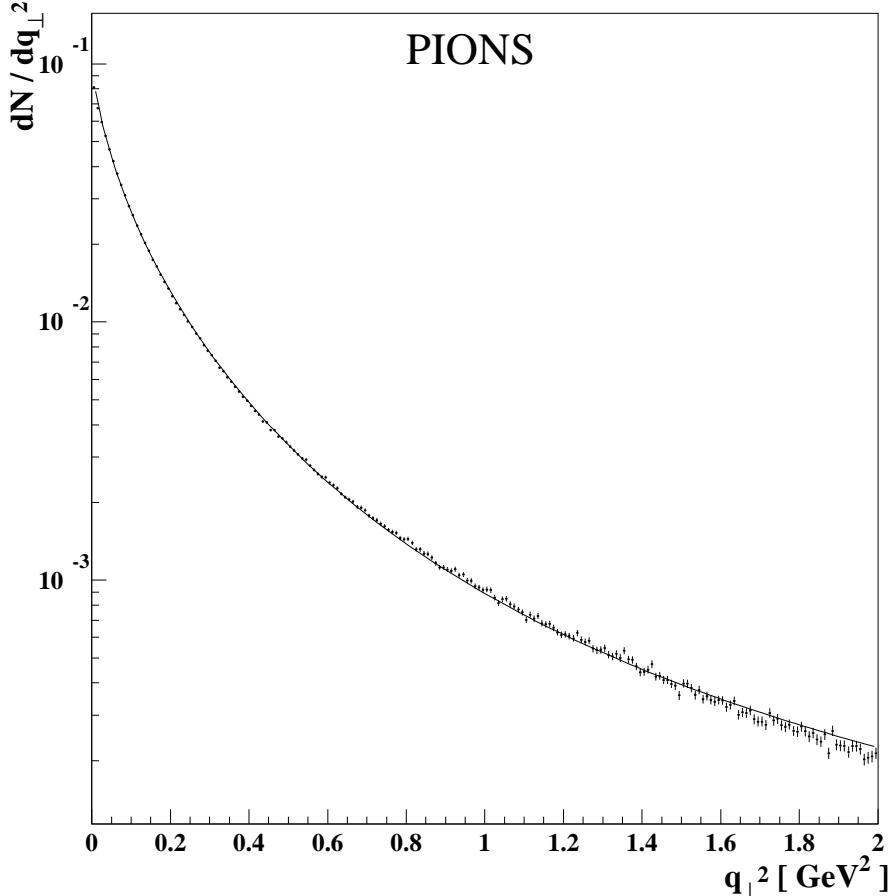


Figure 1: The fit of the formula (26) to q_{\perp}^2 distribution of pions, ($v = 0.286$, $\delta_{\perp} = 0.303$, $\delta_{\parallel} = 0.172$)

6 Two-particle correlation function

The two-particle correlation function is calculated according to formula (29) using the values of the parameters from Table 1. Correlations between parameters determined by the fit are taken into account by sampling the area of the 3σ correlation ellipse for each parameter pair with a Gaussian distribution. At the presence of correlations between the parameters this procedure results in a much

Table 1: Model parameters determined in the fit.

v	δ_{\perp} (GeV)	δ_{\parallel} (GeV)	χ^2/dof
$0.286^{+0.014}_{-0.010}$	$0.303^{+0.010}_{-0.021}$	$0.172^{+0.178}_{-0.015}$	397/193

more reliable estimate of the statistical error on the calculated correlation function value than the 1-dim. errors given by MINUIT. Systematic effects of the fit are accounted for by repeating calculations for extreme values of the parameters allowed by the q_{\perp}^2 distribution. This procedure was needed especially for the parameter δ_{\parallel}^2 which is the least constrained by the one-particle distribution. In this way the value of the correlation function for each particle was determined together with its error, where both error contributions described above have been added linearly. The result of the calculation for pions is shown in Figs.2a,b for the perpendicular and parallel components, respectively.

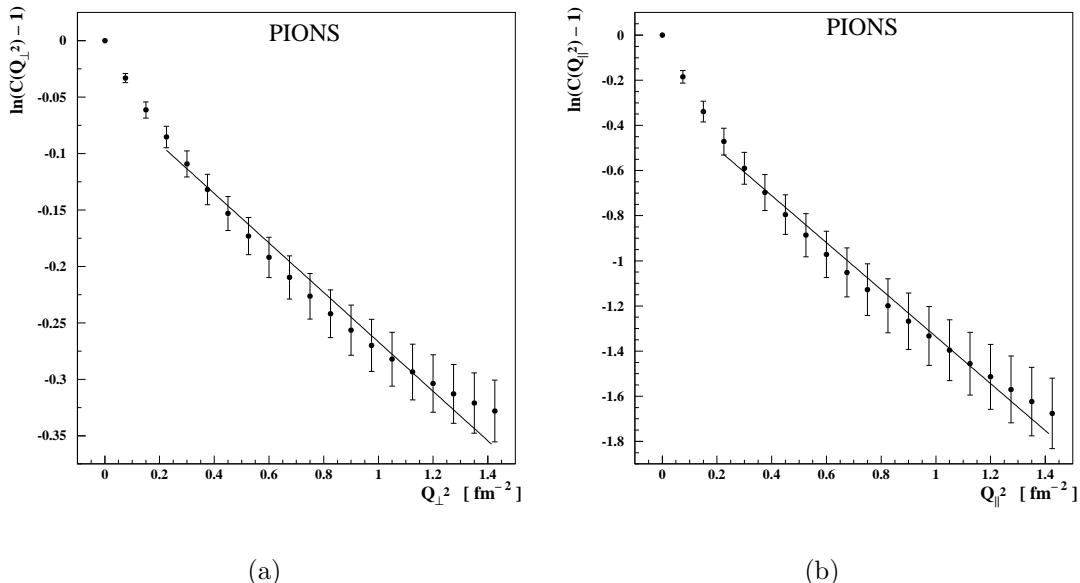


Figure 2: Correlation function vs Q_{\perp}^2 (a) and Q_{\parallel}^2 (b) for π

One sees that these functions are not gaussian. Nevertheless, to compare

with the existing data, we have approximated the results by Gaussians in the region $0.100\text{GeV} < Q < 0.250\text{GeV}$ (Fig. 2). The exclusion of the small Q^2 region may be justified by the well-known fact that the measurements of the correlation function at very small Q^2 are very uncertain (and often this region is omitted in the data analyses at LEP). The results of this procedure for pions are shown in Fig. 3(a) together with measurements of the three LEP experiments [3] done in correlation studies in two- and three dimensions.

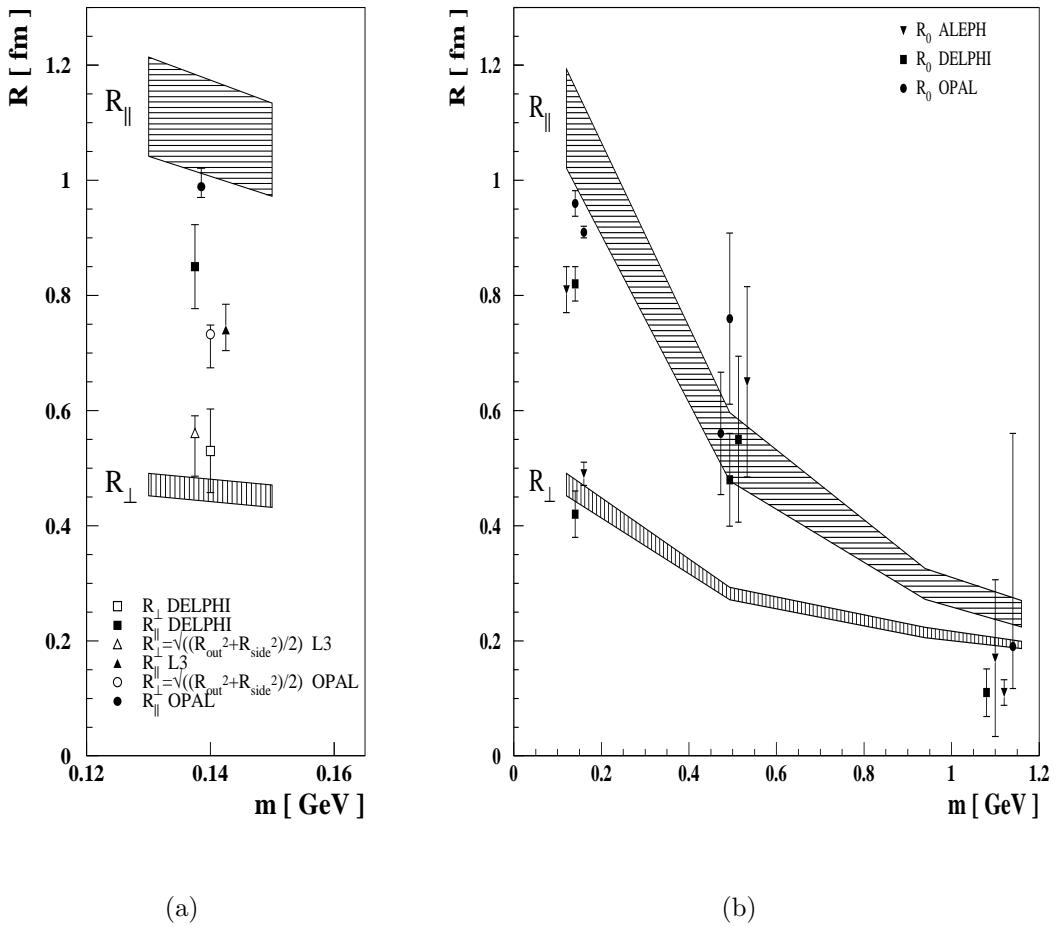


Figure 3: R_{\parallel} and R_{\perp} calculated from the model (shaded bands) : (a) for π , (b) for π, K, p, Λ . Data points in (a) represent results of 2- and 3-dimensional analyses of LEP data [3]. Data points in (b) represent 1-dim source radius R_0 [4]

In the experimental studies Q^2 is decomposed into three components in the longitudinal center-of-mass (LCMS) frame, where the sum of the three-vector

momenta is perpendicular to the thrust axis. Choosing Q_L parallel to the thrust axis, Q_{out} parallel to the sum of the momenta (in the LCMS frame) and Q_{side} orthogonal to both, the decomposition reads:

$$Q^2 = Q_L^2 + Q_{side}^2 + Q_{out}^2(1 - \beta^2); \quad \beta = \frac{p_{out}^1 + p_{out}^2}{E_1 + E_2} \quad (36)$$

DELPHI performed the analysis in two dimensions, determining correlation radii corresponding to $Q_T^2 = Q_{side}^2 + Q_{out}^2$ (R_\perp) and to Q_L^2 (R_\parallel). The L3 and OPAL analyses were done in three components, therefore, as the measure of R_\perp we have chosen $R_\perp = \sqrt{\frac{R_{out}^2 + R_{side}^2}{2}}$. The spread of the experimental results is considerable, which might be due to different methodology (eg. different reference sample, corrections for Coulomb repulsion etc) and differing phase space regions selected. However the clustering of the data points for R_\parallel at larger values than those for R_\perp is clearly visible. Shaded bands in the plot represent the errors on the calculated R_\perp and R_\parallel .

The most remarkable feature seen in this figure is that the model predicts, in agreement with experiment, a longitudinal radius much larger than the transverse one. One also sees that both radii fall in the ballpark of about 1 fm which is hardly surprising. It is also clear that -within the rather large theoretical and experimental errors- the model seems to account for the gross features of the data.

The calculations were also done for kaons, protons and Λ 's. Since the available data samples for these particles are much smaller than those for pions, their q_\perp^2 distributions are not discriminative enough to pin-point reliably the model parameters. Therefore the parameters were taken the same as for pions (given in Table 1). This assumption is to be verified once better data are available but we have checked that it reproduces reasonably well the main characteristics of the transverse momentum distributions of kaons, protons and Λ 's. The resulting correlation functions for all these particles are of Gaussian form over a wide range of Q_\perp^2 and Q_\parallel^2 (up to several hundred MeV) and thus the source radii are well defined.

The mass dependence of the calculated R_\parallel and R_\perp is plotted in Fig. 3(b) where also the available results of the LEP experiments [4] are shown.

One sees that the inequality $R_\parallel > R_\perp$ is still satisfied in the model, although the difference between the two radii at higher masses is not as large as in the case of pions. The data points in this figure correspond to the correlation radius R_0 determined in 1-dimensional analyses (the only available data for heavy particles)[4]. Multiple entries of the result from the same experiment for pions correspond to different measurements made with different reference samples. The points at kaon mass represent measurements for both K_s^0 and K^\pm pairs. The measurements for Λ pairs come from spin analysis (except for the second ALEPH point with small error) where there is no need for a reference sample.

The correspondence between R_0 and the two radii R_{\parallel} and R_{\perp} is not obvious (at least experimentally), but the trend of the data is reasonably well reproduced by the model. More accurate data on kaons would be of great help to further elucidate this point.

7 Conclusions and Comments

In conclusion, we have found that the correlation between the momentum and the production point of a produced hadron, suggested by the Gottfried-Bjorken hypothesis of in-out cascade, seems to account (at least approximately) for the observed correlation between identical particles observed in e^+e^- annihilation. Together with data on the single particle transverse momentum distribution, it predicts strong anisotropy of the two-pion correlation function, in agreement with the observations. The mass dependence of the "effective source radius" is also adequately described. Large uncertainties, both in the theoretical determination of the model parameters, and in the experimental data do not allow, however, to obtain more quantitative conclusions.

Several comments are in order.

(i) One sees from the Table 1 that out of the three parameters determined from transverse momentum distribution, only two (v and δ_{\perp}) are relatively well constrained, whereas δ_{\parallel} is only weakly restricted. It is interesting to note that - within the wide range allowed - the value of δ_{\parallel} is consistent with the relation $\delta_{\parallel} = \delta_{\perp}$ corresponding to the quasi-isotropic case. Accepting this isotropy, we see that the effective width of the momentum distribution around the average given by the Bjorken-Gottfried condition (1) is close to 300 MeV, a rather reasonable value.

The small value of v is also comforting, as it guarantees that the transverse expansion of the "tube" from which the particles are emitted is not unreasonably fast.

(ii) The mass dependence of the effective HBT correlation radii calculated from the model and shown in Fig.6 was obtained under the assumption that the parameters of the model do not depend on particle masses. This assumption was verified (within large experimental uncertainties) for v , δ_{\perp} and δ_{\parallel} . No such direct check is available for τ_0 . However, since the observed mass dependence of the HBT correlation radii does agree -at least approximately - with the experimental observations, we may take it as an argument that *also τ_0 does not depend on particle masses*:

$$\tau_0 \sim \text{const}(M_{\perp}). \quad (37)$$

This seems a rather non-trivial conclusion, as it indicates that -within the Bjorken-Gottfried hypothesis (1)- all particles are emitted at, roughly, the same proper time τ_0 .

This result may be contrasted with a simple expectation (sometimes justified by uncertainty principle) suggesting

$$\tau_0 \sim \frac{1}{M_\perp} \quad (38)$$

which would give a stronger drop of R with increasing mass, particularly for transverse radius.

It seems that also the models based on the picture of string decay [15] would not fulfill the condition (37) but rather obey

$$\tau_0 \sim M_\perp. \quad (39)$$

Such a relation corresponds, in our model, to a much weaker dependence of the correlation radii on the particle masses and thus an other mechanism would have to be invented to describe the data [16].

(iii) In the present paper, studying the two-particle distribution, we considered -following the approach employed in experimental analyses [3]- the boost invariant variable Q_\perp^2 and the variable Q_\parallel^2 evaluated in the longitudinal center-of-mass system. Assuming boost invariance and azimuthal symmetry of the distributions, one finds that a complete analysis would involve 4 variables. As it is convenient to choose them boost invariant, one could use for instance the two transverse momenta $|p_{1\perp}|$ and $|p_{2\perp}|$ the relative azimuthal angle $\phi_1 - \phi_2$ and the relative rapidity $y_1 - y_2$. It would be interesting to see the data analysed in this way.

(iv) Another interpretation of the experimentally observed HBT parameters was given in [17]. The authors take the point of view that the observed HBT radii do indeed correspond to the actual size of the particle emission region which is thus strongly dependent on the particle mass. They argue that this dependence may be understood from the uncertainty principle. It may be worth to point out that this approach is rather different from ours. In our description the parameters characterizing the particle emission region are *mass independent* and the observed change in the HBT radii comes *solely* from the momentum-position correlation as expressed in the assumed Bjorken-Gottfried condition (1).

(v) Our prescription for construction of the two-particle correlation function out of the single-particle source function is similar to -but not identical with- that advocated recently by Heinz and collaborators [7, 18]. The difference is in the treatment of the fourth component of the average momentum vector of the two particles [$\vec{P} \equiv (\vec{q}_1 + \vec{q}_2)/2$]. As seen from (15), we use

$$P_0 \equiv \frac{1}{2}(E_1 + E_2) \quad (40)$$

which guarantees the four-vector character of P_μ but brings it off mass-shell. Heinz et al. propose to take

$$P_0 = \sqrt{m^2 + (\vec{P})^2} \quad (41)$$

which insures $P_\mu P^\mu = m^2$ but induces complicated transformation properties. In practice, however, one is only interested in the region $\vec{q}_1 \approx \vec{q}_2$ where these two prescriptions are not substantially different (the source functions themselves, however, are not the same).

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